INDIAN STATISTICAL INSTITUTE (BANGALORE)
BACK-PAPER EXAMINATION
II SEMESTER, 2009-2010
ANALYSIS IV
INSTRUCTOR: K. RAMA MURTHY

Max. marks: 100 Time Limit: 3hrs

1. Let $[-1,1]^{\infty}$ be the set of all sequences of numbers in $[-1,1]$ with the metric defined by $d(\{a_n\},\{b_n\}) = \sum\limits_{n=1}^{\infty} \frac{1}{2^n} \frac{|a_n - b_n|}{1+|a_n-b_n|}$.

Is this space connected? Is it compact? Find a countable subset of $[-1,1]^{\infty}$ which is dense. (*Prove* that the subset you come up with is dense). [20]

2. Prove that a subset A of a metric space is compact if and only if it is complete and totally bounded. [20]

3. Prove or disprove: The vector space spanned by $1, e^{x^2}, e^{2x^2}, e^{3x^2}, \ldots$ is dense in $C[0,1]$. [15]

4. Prove that there is a continuous periodic function f (with period 2π) such that $\hat{f}(n) = \frac{\log(n)}{n^{3/2}}, n \neq 0, \hat{f}(0) = -1$. [15]

5. Prove without any computation that $\int\limits_0^{\pi} \cos^5(x)\cos(6x)dx = 0$. [15]

6. Prove Weierstrass approximation theorem using cesaro convergence of Fourier series. [15]